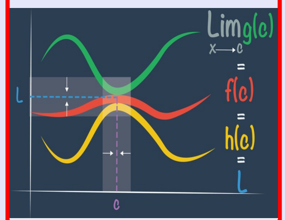
Math 261

Fall 2022

Lecture 36



Introduction to integration:

Anti-Derivative

$f'(x) = 2x$ $\qquad f(x) = ?$

$f(1) = 4$ $\qquad f(x) = x^2 + C$ $\qquad f'(x) = 2x + 0 = 2x$

$f(1) = 1^2 + C = 4$

$\boxed{C = 3}$

$\boxed{f(x) = x^2 + 3}$

$f'(x) = x\sqrt{x}$ $\qquad f'(x) = x^{\frac{3}{2}}$

$f(4) = 6$ $\qquad f(x) = \dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$

Find $f(x)$.

$f'(x) = x^n$ $\qquad f(x) = \dfrac{x^{n+1}}{n+1} + C$ $\qquad f(x) = \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$

$f(x) = \frac{2}{5} x^2\sqrt{x} + C$

$f(4) = \frac{2}{5} \cdot 4^2\sqrt{4} + C = 6$

$= \dfrac{2 \cdot 16 \cdot 2}{5} + C = 6$

$= \dfrac{64}{5} + C = 6$

$C = 6 - \dfrac{64}{5}$

$C = \dfrac{-34}{5}$

$\boxed{f(x) = \frac{2}{5} x^2\sqrt{x} - \frac{34}{5}}$

$f'(x) = 12x^2 - 6x + 8$

$f(0) = 4$

$f(x) = \frac{12x^3}{3} - \frac{6x^2}{2} + 8x + C$

$f(x) = 4x^3 - 3x^2 + 8x + C$

$f(0) = 4(0)^3 - 3(0)^2 + 8(0) + C = 4$

$\boxed{C = 4}$

$\boxed{f(x) = 4x^3 - 3x^2 + 8x + 4}$

$f'(x) = 2\cos x + \sec^2 x$, $f(x) = 2\sin x + \tan x + C$

$f(\pi/3) = 4$

$f(\frac{\pi}{3}) = 2\sin\frac{\pi}{3} + \tan\frac{\pi}{3} + C = 4$

$\cancel{2} \cdot \frac{\sqrt{3}}{\cancel{2}} + \sqrt{3} + C = 4$

$C = 4 - 2\sqrt{3}$

$\boxed{f(x) = 2\sin x + \tan x + 4 - 2\sqrt{3}}$

$f''(x) = 20x^3 - 12x^2 + 6x$

$\boxed{f'(0) = 2}$

$f(0) = -2$

$f'(x) = \frac{20x^4}{4} - \frac{12x^3}{3} + \frac{6x^2}{2} + C$

$f'(x) = 5x^4 - 4x^3 + 3x^2 + C$

$5(0)^4 - 4(0)^3 + 3(0)^2 + C = 2$

$\boxed{C = 2}$

$f'(x) = 5x^4 - 4x^3 + 3x^2 + 2$

$f(x) = \frac{5x^5}{5} - \frac{4x^4}{4} + \frac{3x^3}{3} + 2x + C$

$f(x) = x^5 - x^4 + x^3 + 2x + C$

$f(0) = 0 - 0 + 0 + 0 + C = -2$

$\boxed{C = -2}$

$\boxed{f(x) = x^5 - x^4 + x^3 + 2x - 2}$

$f''(x) = 4 + 6x + 24x^2$

$f(0) = 3$ $\quad f'(x) = 4x + \frac{6x^2}{2} + \frac{24x^3}{3} + C_1$

$f(1) = 10$ $\quad f'(x) = 4x + 3x^2 + 8x^3 + C_1$

$f(x) = \frac{4x^2}{2} + \frac{3x^3}{3} + \frac{8x^4}{4} + C_1 x + C_2$

$f(x) = 2x^2 + x^3 + 2x^4 + C_1 x + C_2$

$f(0) = 0 + 0 + 0 + C_1(0) + C_2 = 3 \rightarrow \boxed{C_2 = 3}$

$f(1) = 2(1)^2 + 1^3 + 2(1)^4 + C_1(1) + 3 = 10$

$2 + 1 + 2 + C_1 + 3 = 10 \quad \boxed{C_1 = 2}$

$\boxed{f(x) = 2x^2 + x^3 + 2x^4 + 2x + 3}$

$f''(x) = 2 + \cos x$

$f(0) = -1$ $\quad f'(x) = 2x + \sin x + C_1$

$f(\frac{\pi}{2}) = 0$ $\quad f(x) = x^2 - \cos x + C_1 x + C_2$

$f(0) = \cancel{0^2}^{\,0} - \cancel{\cos 0}^{\,1} + \cancel{C_1(0)}^{\,0} + C_2 = -1$

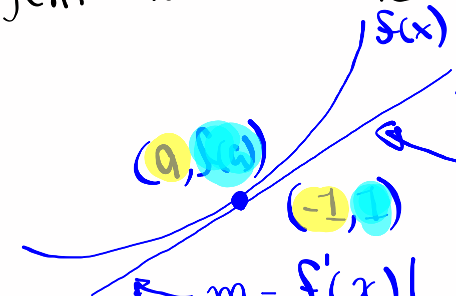
$\boxed{C_2 = 0}$

$f(x) = x^2 - \cos x + C_1 x$

$f(\frac{\pi}{2}) = \frac{\pi^2}{4} - \cancel{\cos\frac{\pi}{2}}^{\,0} + C_1 \cdot \frac{\pi}{2} = 0$ $\quad \frac{\pi}{2} C_1 = -\frac{\pi^2}{4}$ $\quad \boxed{C_1 = -\frac{\pi}{2}}$

$\boxed{f(x) = x^2 - \cos x - \frac{\pi}{2} x}$

Find a function with $f'(x)=x^3$ and tangent to the line $x+y=0$.



$x+y=0$

$y=-x$

$m=-1$

$m=f'(x)\big|_{x=a}$

$-1+y=0$

$y=1$

$f'(a)=-1$

$a^3=-1$

$a=-1$

$f'(x)=x^3$

$f(x)=\frac{x^4}{4}+C$

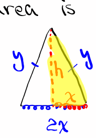
$f(-1)=1$

$\frac{(-1)^4}{4}+C=1$

$\boxed{C=\frac{3}{4}}$

$\Rightarrow \boxed{f(x)=\frac{1}{4}x^4+\frac{3}{4}}$

Show that of all isosceles triangles with a given perimeter $\to 2P$, the one with greatest area is equilateral.



$2P=2x+2y \to P-x=y$

$\text{Area}=\frac{2x\cdot h}{2}=x\cdot h$

$x^2+h^2=y^2 \qquad h^2=y^2-x^2$

$h=\sqrt{y^2-x^2}$

$\text{Area}=x\sqrt{y^2-x^2}$

$A(x)=x\sqrt{(P-x)^2-x^2} \qquad A(x)=x\sqrt{P^2-2Px}$

$A(x)=\sqrt{x^2(P^2-2Px)} \qquad A(x)=(P^2x^2-2Px^3)^{1/2}$

$A'(x)=\frac{1}{\cancel{2}}(P^2x^2-2Px^3)^{-1/2}\cdot(\overset{1}{\cancel{2}}P^2x-\overset{3}{\cancel{6}}Px^2)$

$A'(x)=\frac{Px(P-3x)}{\underbrace{\sqrt{P^2x^2-2Px^3}}_{\text{Non-Negative}}}$

$A'(x)=0 \to \cancel{x=0} \qquad x=\frac{P}{3}$

$+ \quad -$

$\frac{P}{3}$

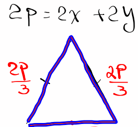
Max.

Max. Area when $x=\frac{P}{3}$

$2P=2x+2y \qquad P=x+y \to y=P-x$

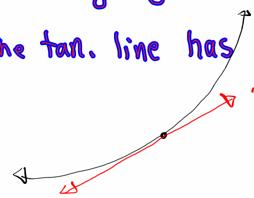
$=P-\frac{P}{3}$

$=\frac{2P}{3}$



$2x=2\left(\frac{P}{3}\right)=\frac{2P}{3}$

At which points on the curve given by $y = 1 + 40x^3 - 3x^5$ the tan. line has the largest slope?



max. slope

$m = f'(x)$

$= 120x^2 - 15x^4$

$m(x) = 120x^2 - 15x^4$ ← Need to maximize

$m'(x) = 240x - 60x^3$

$m''(x) = 240 - 180x^2$

$m''(0) = 240 > 0$ Min.

$m''(2) = 240 - 180(2)^2 < 0$ Max

$m''(-2) = 240 - 180(-2)^2 < 0$ Max

$m'(x) = 0$

$240x - 60x^3 = 0$

$4x - x^3 = 0$

$x = 0$

$x = 2$

$x = -2$

Max. at

$(2, ?)$

$(-2, ?)$

Use original curve, evaluate at 2 & -2 to get